Supersonic Turbulence in the ISM: stellar extinction determinations as probes of the structure and dynamics of dark clouds

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ABSTRACT

Lada et al. (1994) have described a method for studying the distribution of dust in dark clouds using infrared imaging surveys. In particular they show that the method provides some information about the structure of the gas (dust) on scales smaller than their resolution.

In the present work we clarify the nature of the information provided by their method.

We show that:

- the 3D density field of the gas is well described by a Log-Normal distribution down to very small scales;
- the power spectrum and the standard deviation of the 3D density field can be constrained;
- the origin of such a structure of the density field is likely to be the supersonic turbulence in the gas.

In fact we find a qualitative and quantitative agreement between the predictions based on recent numerical simulations of supersonic turbulence (Nordlund and Padoan 1996; Padoan, Nordlund and Jones 1996) and the constraints given by the infrared dust extinction measurements.

Subject headings: ISM: extinction - kinematics and dynamics

1. Introduction

In a recent paper Lada et al. (1994) have illustrated the method of mapping the distribution of dust, and therefore gas, in dark clouds by using stellar extinction measurements in the near-infrared. The method is based on the use of multi-channel array cameras that allow the simultaneous determinations of the colors of hundreds to thousands of stars through a molecular cloud. The infrared color excess is proportional to the dust column density, and the dust-to-gas ratio is known to be nearly constant in interstellar clouds; therefore cloud maps can be obtained. The maps obtained measuring the infrared excess are considerably more accurate than the maps based purely on stellar counts.

The gas column density is given by

$$N(H+H_2) = 2 \times 10^{21} A_V cm^{-2} \tag{1}$$

where the visual extinction in magnitudes is

$$A_V = 15.9 E(H-K) \tag{2}$$

and the color excess is

$$E(H-K) = (H-K)_{observed} - (H-K)_{intrinsic} \tag{3}$$

and finally

$$< (H-K)_{intrinsic} >= 13 \pm 0.01 mag. \tag{4}$$

(see equations 1-4 in Lada et al. 1994).

The extinction data are used in two complementary ways, one exploiting an ordered sampling (information on large scale structure), the other a random sampling (information on scales below the resolution of the map).

In the first method of analysis the data are spatially binned like in stellar count and millimeter wave observations. At any position a few stars are found so that an average extinction A_V can be measured. The result is an extinction map that compares well with the stellar count map and the CS map.

The second method is that of plotting the mean extinction, A_V, and its standard deviation, σ, measured at any position. Lada et al. (1994) found that the dispersion grows with the average extinction, and realized that this behavior contains information about the structure of the extinction (therefore of the gas mass distribution) in the cloud, on scales smaller than the resolution of the extinction map. They give

examples of mass distributions that would generate or not generate such a plot, but their interpretation of the plot does not go very far.

In this work we focus on the second method of using the extinction data, that is on the meaning of the $\sigma - A_V$ plot as a tracer of structure on scales below the resolution of the map.

Fig.1, which is the equivalent of fig.7 in Lada et al. (1994), shows the $\sigma - A_V$ plot obtained from the original data, kindly provided to us by the authors. The measurements are taken for a dark cloud complex near the young cluster IC 5146 in Cygnus. The cloud has been mapped in ^{12}CO and ^{13}CO by Dobashi et al. (1992), who named it 'Cloud C'.

In sections 3 and 4 we show that the $\sigma - A_V$ plot is due to the 'intermittent' distribution of the dust (that is of the gas density field in the cloud), and we show how to constrain such distribution using randomly generated fields with given statistics and power spectra. Before giving such details, though, we present in the next section the results of recent numerical simulations, concerning the density field in isothermal random supersonic flows. It will be clear, in section 4 and in the following discussion, that random supersonic flows are in fact excellent candidates to interpret the extinction data and to explain the origin of the distribution of dust in dark clouds.

2. Supersonic Turbulence

Nordlund and Padoan (1996) and Padoan, Nordlund and Jones (1996) have recently discussed the importance of supersonic flows in shaping the density distribution in the cold interstellar medium (ISM).

They have run numerical simulations of isothermal flows randomly forced to high Mach numbers. Their experiments are meant to represent a fraction of a giant molecular cloud: $\approx 10pc$ in size and $10^3 M_\odot$ in mass. The simulated random supersonic motions are in fact observed in molecular clouds.

It is found that most of the mass concentrates in a small fraction of the total volume of the simulation, with a very intermittent distribution. The probability density function (pdf) of the density field is well approximated by a Log-Normal distribution:

$$P(lnx)dlnx = \frac{1}{(2\pi\sigma_{lnx}^2)^{1/2}} exp\left[-\frac{1}{2}\left(\frac{lnx - \overline{lnx}}{\sigma_{lnx}}\right)^2\right] \qquad (5)$$

where x is the relative number density:

$$x = n/\overline{n} \tag{6}$$

and the standard deviation σ_{lnx} and the mean $\overline{lnx}$ are functions of the rms Mach number of the flow, $\mathcal{M}$:

$$\overline{lnx} = -\frac{\sigma_{lnx}^2}{2} \tag{7}$$

and

$$\sigma_{lnx}^2 = ln(1 + \frac{\mathcal{M}^2 - 1}{\beta}) \tag{8}$$

or for the linear density:

$$\sigma_x = \beta(\mathcal{M}^2 - 1)^{0.5} \tag{9}$$

where $\beta \approx 0.5$. Therefore the standard deviation grows linearly with the rms Mach number of the flow.

It is also found that the power spectrum, $P(k)$, of the density distribution is consistent with a power law:

$$P(k) \sim k^{-2.6} \tag{10}$$

where k is the wavenumber.

We will show in the following sections that the extinction data are consistent with these theoretical predictions.

3. Numerical Generation of Extinction Determinations

In order to interpret the extinction data we have generated random 3D density distributions with given statistics and power spectra, projected them in 2D, and sampled them randomly as it happen when stars are found through the cloud. The stars are assumed to be uniformly distributed in space. Then a grid has been created on the distribution and the mean extinction, A_V, and its dispersion, σ, have been measured in every bin using the position selected by the few 'stars' found in the bin.

In fig.2 we show the case of a Gaussian distribution, to be compared with the case of a Log-Normal distribution, shown in fig.3. It is only in the case of the Log-Normal distribution that the plot $\sigma - A_V$ is

similar to the observational one (fig.1). Clearly some sort of intermittent tail is needed in order to produce the growth in the dispersion with the growth in mean extinction.

Intermittency is a natural explanation of the plot and it is also the main feature of the density distribution in supersonic turbulence. Note that having only high density clumps (e.g. steep power spectrum) is not enough to generate the plot, as already shown by Lada et al. 1994.

We have studied the sensitivity of the plot to different power spectra and standard deviations in the 3D density distribution. This is an important point because the power spectrum cannot be measured accurately at the moment in the numerical simulations of supersonic turbulence, since it requires a huge dynamical range, and therefore it is interesting to find constraints for it via observations of the projected density field in dark clouds with supersonic random motions. The standard deviation is instead measured in the numerical simulations as a function of the rms Mach number of the flow, $\mathcal{M}$, and may be directly compared with the observed one, if the rms velocity in the cloud is measured, as is done by millimeter wave observations.

Fig.4 shows a $\sigma - A_V$ plot from a random distribution with standard deviation larger than the in the case of fig.3: the slope of the plot increases together with the standard deviation of the density field. This behavior is easily understood. In fact the plot is related to the structure of the density field on a scale below the resolution of the extinction map: if there were no structure on such small scale, σ would be close to zero. The larger the fluctuations on small scale, the larger σ.

4. Statistics and Power Spectrum of the ISM Density Field

Fig.1 shows the observational $\sigma - A_V$ plot. A linear regression analysis gives:

$$\sigma = const + (0.35 \pm 0.01) A_V \tag{11}$$

where the value of the constant is irrelevant in the present work, because the numerical version of the plot can be freely translated along A_V. Note that even the values $\sigma = 0.0$ due to the presence of only a single star in the bin are used. The elimination of those values would give the linear regression coefficient found by Lada et al. (1994).

We want to understand now how the linear regression is affected by the errors in the color excess. Lada et al. (1994) estimated a maximum error in the color excess of ± 0.15 mag, that translates into an error of

2.5 mag in A_V.

In order to study the effect of the color excess errors we have randomly added such errors to the original data, both with a normal and with a uniform distributions. This can be done many times, until any correlation $\sigma - A_V$ is completely lost. By applying the errors once, we find on the average a coefficient (0.33 ± 0.02). This is an encouraging result, because it means that, even after the addition of the errors, the uncertainty in the coefficient is still low. Of course the coefficient has decreased a bit, because the correlation between σ and A_V is diminished every time errors are added.

The $\sigma - A_V$ relation to be compared with the numerical ones is therefore:

$$\sigma = const + (0.36 \pm 0.02) A_V \tag{12}$$

4.1. Statistics

As we mentioned in the previous section the slope of the numerical plot depends on both the power spectrum index, α, and the standard deviation of the 3-D density distribution, $\sigma_{x,3D}$. We can therefore draw lines of constant $\sigma - A_V$ linear regression coefficient, C_r, on the plane $\alpha - \sigma_{x,3D}$, as shown in fig.5.

Since the value of C_r is known observationally with very small uncertainty (see (12)), its contours on the numerical plane $\alpha - \sigma_{x,3D}$ may in principle be used to constrain the power index of the 3-D density field when its standard deviation is known, or vice-versa.

In fact fig.5 shows that the lines of constant C_r are almost lines of constant $\sigma_{x,3D}$, which means that the plane $\alpha - \sigma_{x,3D}$ is can constrain the value of $\sigma_{x,3D}$, but not that of α. Given the observational value of C_r and the value of α determined below, one gets:

$$\sigma_{x,3D} = 5.0 \pm 0.5 \tag{13}$$

This is the value of the standard deviation of the 3-D density distribution in the ‘Cloud C’, as given by stellar extinction measurements.

4.2. Power spectrum

The $\alpha - \sigma_{x,3D}$ plane does not constrain directly the index of the (power law) power spectrum of the 3-D density distribution. Nevertheless it is indirectly useful because it gives the 3-D standard deviation, that can be compared with the observed 2-D standard deviation, in order to constrain the spectral index. In fact the projection into 2-D of the 3-D distribution is such that the two standard deviations are related in a way that depends on the value of the power spectrum index. This can be shown using the numerically generated random distributions. In fig.6, lines of constant spectral index are plotted in the plane $\sigma_{x,2D} - \sigma_{x,3D}$. For a fixed 3-D standard deviation, the value of the projected 2-D standard deviation decreases towards steeper spectra.

The 2-D standard deviation, $\sigma_{x,2D}$, is measured in the extinction map, on a regular grid that contains on average about 5 stars per bin. Its standard deviation is:

$$\sigma_{x,2D} = 0.7 \pm 0.1 \tag{14}$$

Entering the plane $\sigma_{x,2D} - \sigma_{x,3D}$ with this value and with the previously determined value of $\sigma_{x,3D}$, one gets:

$$\alpha = -2.6 \pm 0.5 \tag{15}$$

This is the value of the power index of the 3-D density distribution in the 'Cloud C'.

5. Discussion

We have seen that the origin of the $\sigma - A_V$ plot is the intermittency in the 3-D density distribution of the dark cloud; i.e., the occurrence of huge density fluctuations with a significant probability. We were inspired towards this explanation of the plot by recent results emerging from our numerical experiments of highly supersonic turbulence. The numerical experiments showed that most of the mass concentrates in a small fraction of the total volume, that very large (orders of magnitude) density contrasts appear in the flow, that the distribution of mass density is well described by a Log-Normal, and that the standard deviation of the statistics grows linearly with the rms Mach number of the flow.

Nevertheless the extinction map itself shows that the distribution of the projected density has an

intermittent tail that resembles a Log-Normal, and even the distribution based on the sampling of column density star by star, which is a random sampling, is qualitatively the same.

The problem here is a typical one in astronomy: extracting whole 3-D fields from their integrated 1-D (eg velocity fields at any point) or 2-D (eg scalar fields in space) counterparts that are observed from our fixed point of view. One way to do this is to build a model for the 3-D field and simulate the observational procedure on that field. The result of the 'numerical' observation is compared with the actual observation, and one may conclude whether or not the model field is consistent with the observation. This method is good only as far as the observations are difficult to reproduce, that is as far as there is no more than one reasonable model that fits the observational data.

We have shown how the generation of random fields with given statistics and power spectra (assumed to be power laws) leads to the plot of contours of constant value of the slope of the $\sigma - A_V$ relation, on the plane $\alpha - \sigma_{x,3D}$, and how one may extract both standard deviation and spectral index of the original 3-D field of the dark cloud.

Clearly *the contours of C_r on the plane $\alpha - \sigma_{x,3D}$, obtained with numerically generated random distributions, is a powerful tool to investigate the 3-D structure of a dark cloud, down to scales smaller than the resolution of the extinction map, when stellar extinction determinations through the cloud are available.*

If it is assumed that the power spectrum is a universal property of turbulence in this regime (highly supersonic and super-Alfvénic, isothermal equation of state), that its shape is in fact a power law, and that dark clouds are in fact in such a turbulent state (as this work indicates) then the observational constraint on the power spectrum obtained in the present work is a prediction that numerical simulations will be able to check more firmly in a few years, when larger numerical simulations (N=500^3-1000^3) will be available. In the meanwhile, this prediction is very useful for order to model the origin of the mass distribution of protostars, given the statistics and the power spectrum of the density field, in a similar way as it is done in cosmology for predicting the mass distribution of galaxies (Press & Schechter 1974). This has in fact been done with considerable success (Padoan, Nordlund, & Jones 1996).

It should be noted that the method of using dust extinction measurements in order to constrain the 3-D density field of dark clouds has several advantages, when compared with the traditional method of using maps of molecular emission lines.

First of all it is a much smaller observational effort.

Secondly the translation of the flux at one given line of one given molecule into column density through the whole cloud is far more complex than the transformation of stellar color excess into column density.

Finally the random sampling of points in space (random locations of single stars behind the cloud) allows for the extraction of information from scales smaller than the resolution of the extinction map, based on averaging at any position a few of the randomly selected points. The information is extracted by using the observational $\sigma - A_V$ plot, together with the numerical prediction of the slope of that plot in the $\alpha - \sigma_{x,3D}$ plane.

The extra information one gets can be summarized in the following points:

- there must be structure on scales at least ten times smaller than the resolution of the extinction map, that is down to 0.02 pc.
- the density distribution is consistent with a Log-Normal
- standard deviation and power index are measured

The present work suggests that the dust extinction measurements are consistent with a scenario where the origin of the complex density field in dark clouds is supersonic turbulence, or more generally the presence of supersonic motions in dark clouds, whatever their origin might be. Such a scenario is appealing, for the simple reason that supersonic motions have indeed been observed and measured in dark clouds for the last twenty years!

The connection between the observations and supersonic turbulence is not only qualitative in nature (Log-Normal shape of the 3-D density distribution), but also quantitative.

In fact, in our numerical experiments (Nordlund & Padoan 1996) we have determined the relation between the standard deviation of the 3-D density field, $\sigma_{x,3D}$, and the rms Mach number of the flow, $\mathcal{M}$ cf. Eq. (9). The same may be done observationally, since Cloud C has been studied in some details by Dobashi et al (1992) in ^{12}CO and ^{13}CO. They obtain measurements of temperature and velocity dispersion in the three main cores, C1, C2 and C3, that are included in the area where stellar extinction is measured by Lada et al. (1994). Using those values one gets $\mathcal{M} \approx 10$. Given the value $\sigma_{x,3D} = 5$ obtained in the present work, one sees that the relation between standard deviation of the density field and rms Mach number of the flow is consistent with the one predicted in our numerical experiments of supersonic turbulence.

Moreover, the spectral index estimated from our simulations, ~ -2.6, is consistent with that obtained from the observational data (see fig.7).

The values of the standard deviation and of the spectral index are useful for modeling the origin of the mass distribution of protostars, given the statistics and the power spectrum of the density field, in a similar way as it is done in cosmology for predicting the mass distribution of galaxies (Press & Schecther 1974). This has in fact been done with considerable success (Padoan, Nordlund, & Jones 1996).

6. Conclusions

In the present work we have re-interpreted the observational results obtained by Lada et al. (1994), i.e. the fact that the mean stellar extinction at any given position in space increases together with the dispersion of the extinction, where the averages are taken among the stars found at that position in space.

The authors were able to conclude that:

- structure must be present down to scales smaller that the extinction map resolution;
- generic models for the cloud structure (eg uniform or in clumps) do not easily reproduce the $\sigma - A_V$ plot;
- the $\sigma - A_V$ plot is a basic test for any model for the dynamics and structure of the cold interstellar medium.

We have simulated the observations by generating random density distributions. In this way we have been able to better define the information contained in the $\sigma - A_V$ plot. We can therefore add that:

- the statistics of the 3-D density field in the dark cloud is certainly very intermittent; in particular it is consistent with a Log-Normal distribution;
- the standard deviation of the statistics is $\sigma_{x,3D} = 5.0 \pm 0.5$;
- the index of the power spectrum (assumed to be a power law), of the 3-D density field, is $\alpha = 2.6 \pm 0.5$;
- the relation between the rms Mach number of the flow and the standard deviation of the 3-D density field is about $\sigma_{x,3D} \approx 0.5\mathcal{M}$;

We therefore conclude that the scenario for star formation and for the dynamics of dark clouds proposed by Padoan (1995), Nordlund & Padoan 1996), and Padoan, Nordlund, & Jones (1996) is fully consistent with the dust extinction measurements in 'Cloud C' by Lada et al. (1994).

In that scenario the dynamics of dark clouds is characterized by supersonic random motions, which are responsible for fragmenting the mass distribution. In the cited works we showed that a MIller-Scalo stellar mass function is a natural consequence of that scenario. Here we have shown that the shape, the standard deviation, and the spectral index of the density distribution, predicted with numerical simulations of supersonic turbulence and used in that scenario, are consistent with the observations of stellar extinction.

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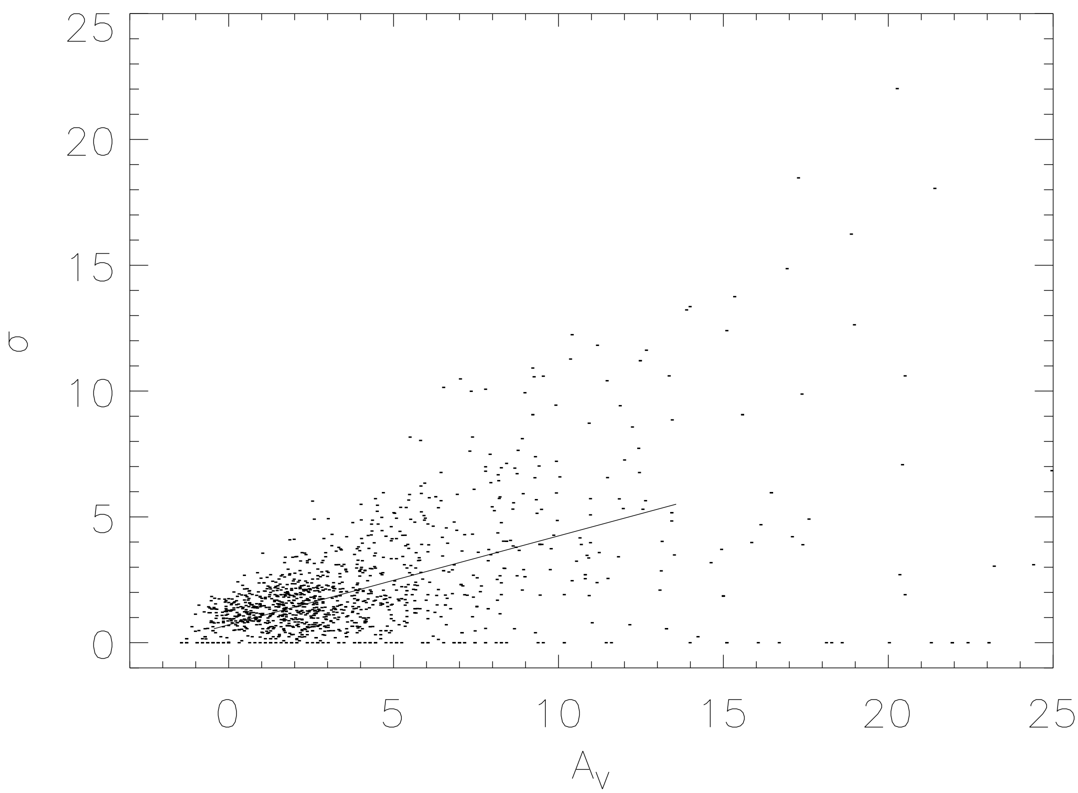

Fig. 1.— The dispersion versus the mean extinction for every bin in the regular grid superposed to the observed region. A bin contains on average about 5 stars. The data are the original ones from Lada et al. (1994)

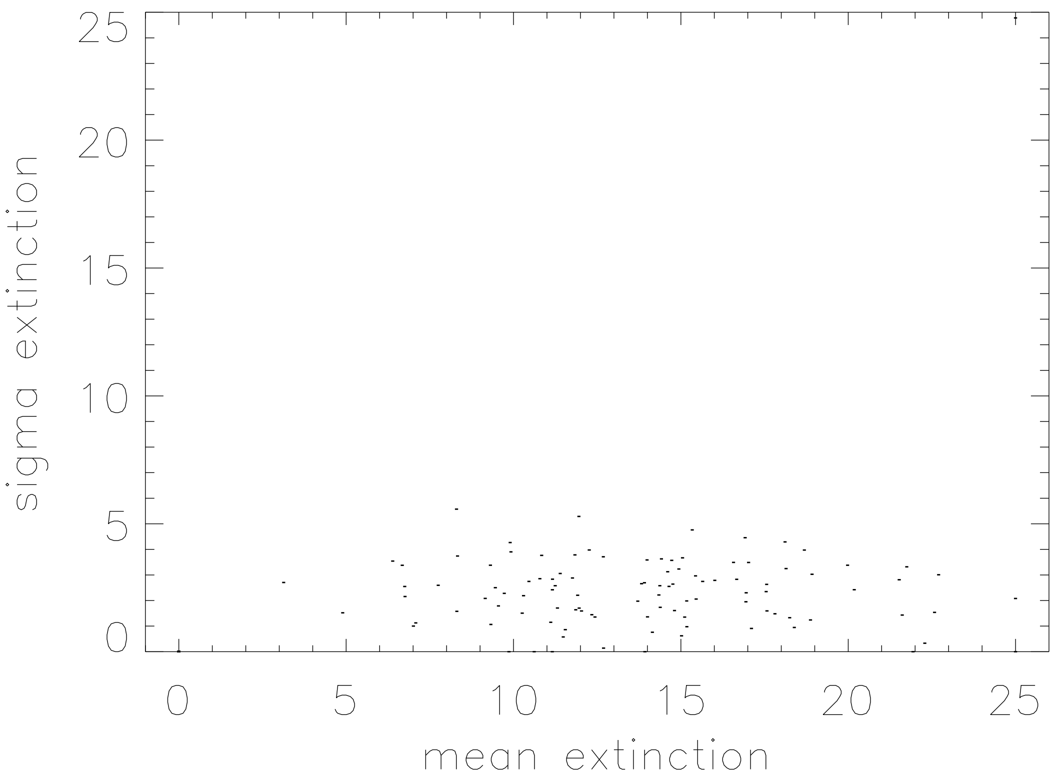

Fig. 2.— The same plot as in fig.1, but obtained numerically starting from a 3-D random distribution with a Gaussian statistic. The Gaussian statistic is clearly unable to reproduce the observed growth of dispersion with mean extinction.

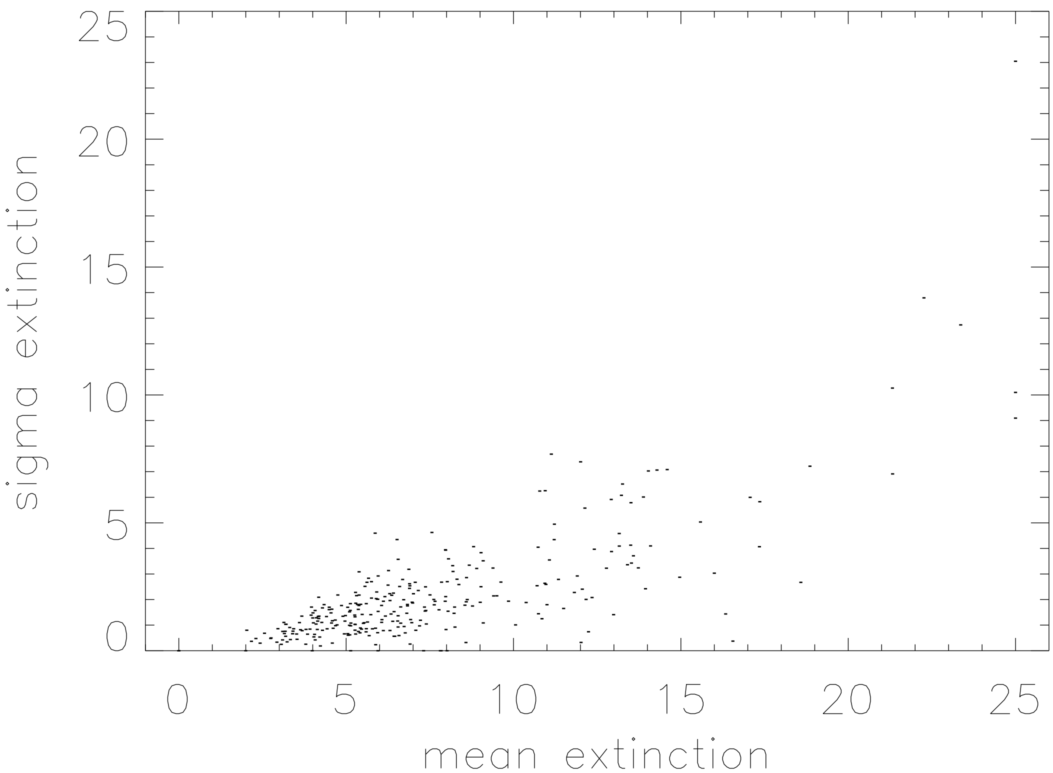

Fig. 3.— The same as in fig.2, but from a Log-Normal distribution: now the observational trend is reproduced.

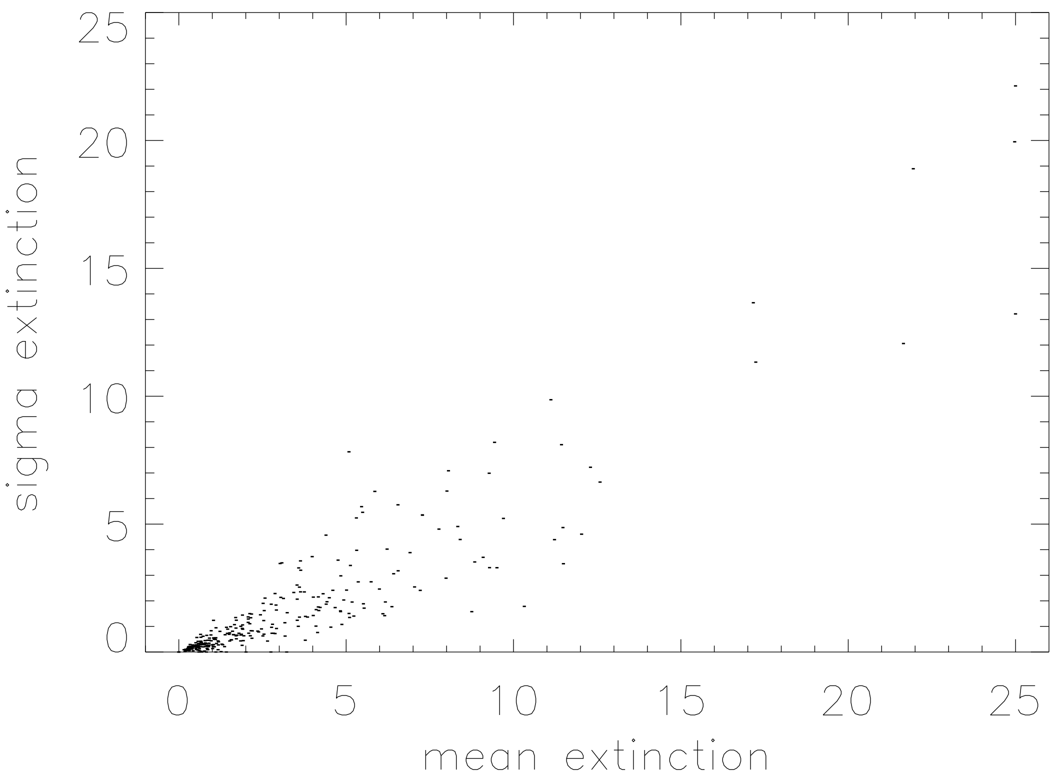

Fig. 4.— The same as in fig3, but with larger standard deviation of the 3-D Log-Normal density distribution.

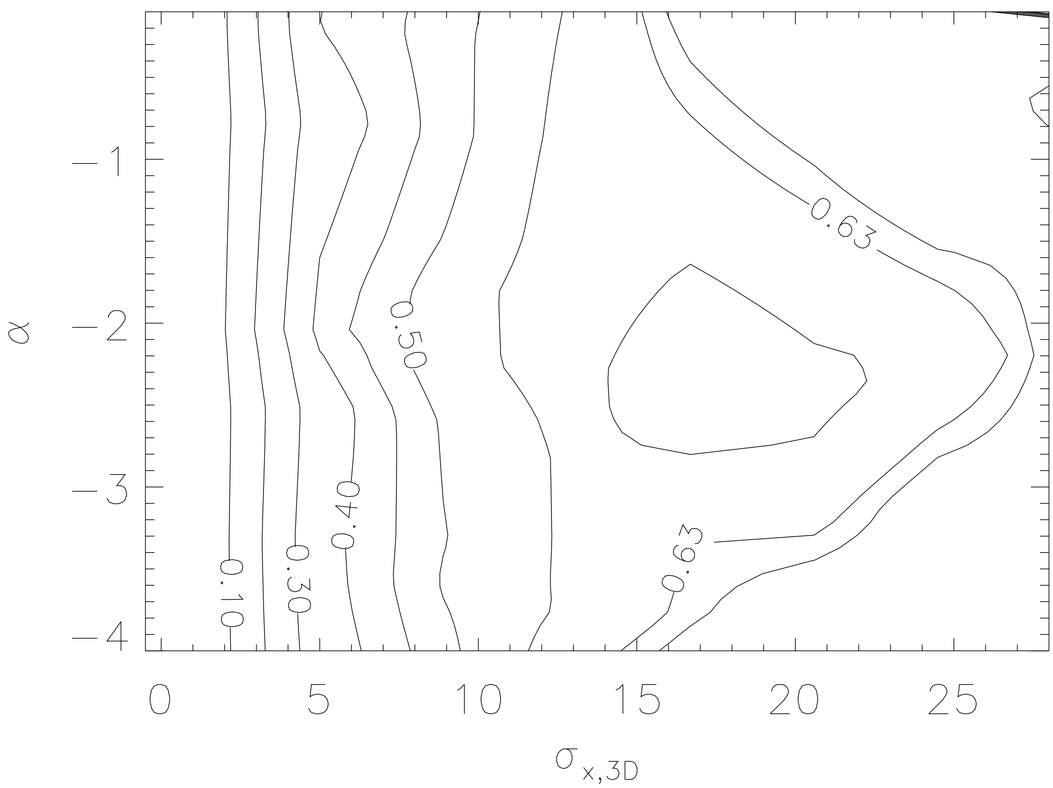

Fig. 5.— Contours of constant value of the slope of the $\sigma - A_V$ plot, C_r. α is the spectral index of the 3-D density distribution (the power spectrum is assumed to be a power law), and $\sigma_{x,3D}$ is the standard deviation of the same distribution. Around the observed value $C_r = 0.36$, the plane gives a good constraint for the 3-D standard deviation, $\sigma_{x,3D}$.

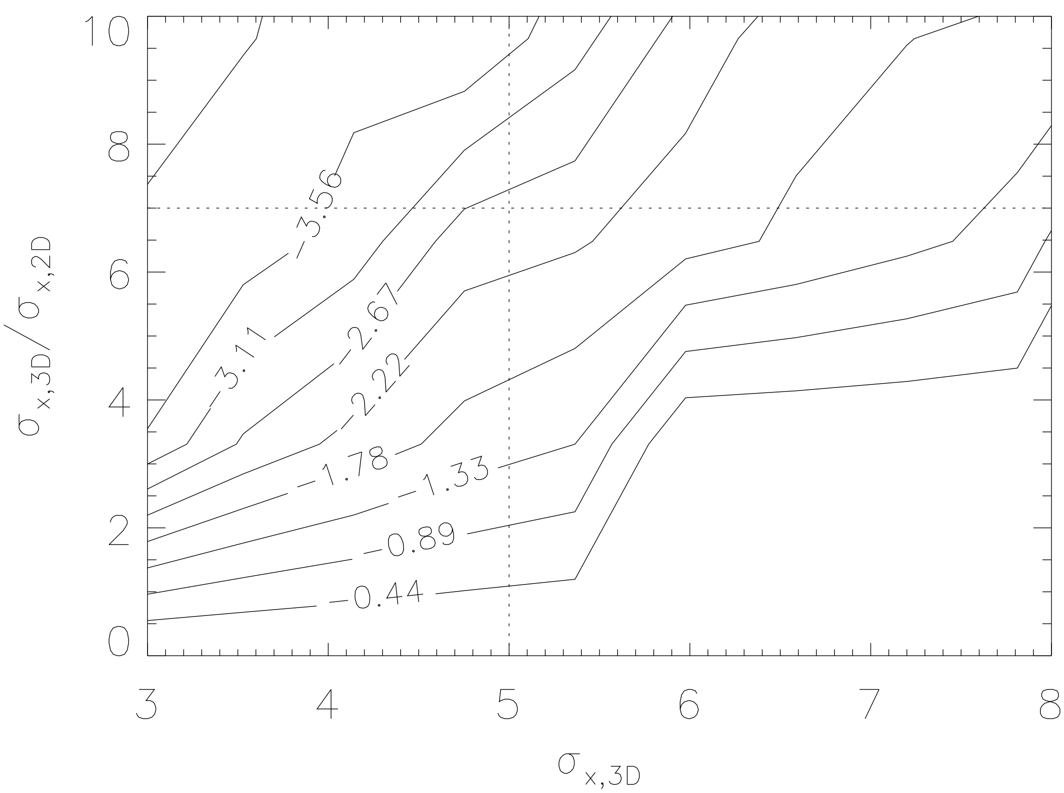

Fig. 6.— Contours of constant spectral index. $\sigma_{x,2D}$ and $\sigma_{x,3D}$ are respectively the 2-D and 3-D standard deviation of the density field.

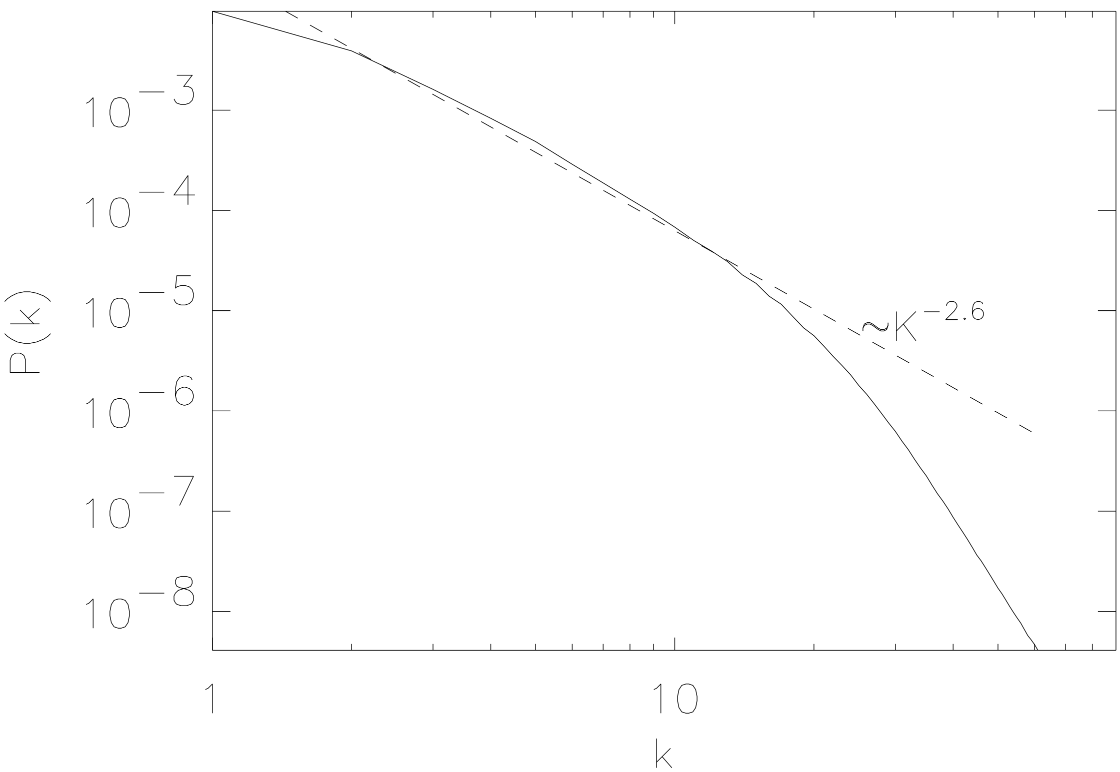

Fig. 7.— The power spectrum in 128^3 simulations of supersonic turbulence (Nordlund & Padoan 1996). The spectrum is consistent with the index $\alpha = -2.6$ inferred from the observations